Complex variable.

Domain (Region) :→

A set of points in the Argand plane is said to be connected set if any two of its points can be joined by a continuous curve, all of whose points belong to S.

Contour :→

By contour, we mean a continuous chain of a finite number of regular arcs.

If the contour is closed and does not intersect itself then it is called closed contour.

Example Boundaries of circle, Δ^s and rectangle.

Cauchy's theorem :→ (Remember)

If a function $f(z)$ is analytic and single valued inside and

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If $f(z)$ is an analytic function of z and if $f'(z)$ is continuous at each point within and on a closed contour C, then

$$\int_C f(z)\,dz = 0$$

where C is any closed contour contained in D

simple closed contour C, then (2)

$$\int_C f(z)\,dz = 0$$

where C is any closed contour contained in D.

Proof → In the proof of this theorem we will apply Green's theorem for a plane which states:

if $P(x,y)$, $Q(x,y)$, $\frac{\partial P}{\partial y}$, $\frac{\partial Q}{\partial x}$ are all (without break) continuous functions within a domain D and if C is any closed contour in D, then

$$\int_C (P\,dx + Q\,dy) = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy$$

Now, $\int_C f(z)\,dz = \int_C (u\,dx - v\,dy) + i\int_C (v\,dx + u\,dy)$ — (1)

we have

$$f'(z) = u_x + i v_x = v_y - i u_y \quad \text{— (2)}$$

[By Cauchy's-Riemann eq's]

Here $f'(z)$ is continuously differentiable
So from (2) u_x, u_y, v_x, v_y all exist and are continuous in D.
Thus from Green's theorem
From (1)

$$\int_C f(z)\,dz = \iint_D \left(-\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) dxdy$$

$$+\, i \iint_D \left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right) dxdy$$

$$= \iint_D \left(-\cancel{\frac{\partial v}{\partial x}} + \cancel{\frac{\partial v}{\partial x}}\right) dxdy$$

$$+\, i \iint_D \left(\cancel{\frac{\partial u}{\partial x}} - \cancel{\frac{\partial u}{\partial x}}\right) dxdy$$

[By Cauchy's Reimann equations]

0 Proved